

# Implication and Lattices

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The chief purpose of this paper is to show that the theory of positive implication algebras (Rasiowa) alias Hilbert algebras (Diego) alias (in dual form) implicative models (Henkin) alias finitely deductive implication algebras (Vukomanovic) provides a complete characterization of intuitionistic implication, as defined on Heyting algebras. This is accomplished by showing that every positive implication algebra can be embedded into a dual symmetrical Heyting algebra (of implicative filters of the given algebra) dually preserving intuitionistic implication, thus yielding completeness. Moreover, such a symmetrical Heyting algebra is dually isomorphic to itself. The same applies, *mutatis mutandis*, to the theory of S4 implication algebras which provides a complete characterization of S4 implication as defined on finitely deductive lattices. That is, every S4 implication algebra can be embedded into a dual finitely deductive lattice dually preserving S4 implication.

## References

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